Operational Risk Capital and Insurance in Emerging Markets

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Resumen

In its first part, this paper develops ideas regarding the importance of the insurance industry as a powerful provider of services to operational risk management, especially in emerging-market economies. The insurance industry not only has the role of financing losses, but also acts as a shadow supervisor of their insured financial institutions. Furthermore, the insurance industry is an operational risk data source and a contributor of techniques to estimate operational risk capital charges. The second part of the paper develops a simple analytical way of computing operational risk capital under the introduction of traditional insurance policies. The methodology is based on Loss Distribution Approaches advanced in the literature of operational risk measurement; however, explicit multivariate analysis is performed in order to capture the richness of the data source that might become available under the proposal made in the first part of the paper. The approach developed in the second part incorporates insurance taking to mitigate specific operational risk categories. This is of potential use by financial institutions, insurers and regulators in order to estimate operational risk losses and the corresponding capital charge.

CLASIFICACION JEL: C15, C46, C63, G21, G22, G28

CLAVE: AMA, Operational Risk, Insurance

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Introduction

For the first time ever, the Basel Committee on Banking Supervision (BCBS) requires financial institutions to hold operational risks capital. At the same time, the revised Framework (given by the BCBS), called Basel 2, opens the possibility to take insurance in order to reduce capital requirements. This procedure is possible as long as the capital requirement and the insurance effect upon it have been calculated by an Advanced Measurement Approach (AMA). The quantitative and qualitative measurements implied by AMA demand the use of internal and external data which must consider the losses, the key business environment and the internal control factors financial institutions face.

Given that emerging economies in different regions of the world will freely adopt Basel II guidelines\(^1\) including the implementation of AMA in the coming ten years, there is a need of appropriate data sources for the particular emerging-market environments. The existing data sources like the Operational Riskdata eXchange Association (ORX) are suited for its large international financial institution members but would not be suitable for financial institutions in emerging-market economies.

In view of the aforementioned issues, this paper has two contributions. First, it highlights the benefits of insurance for the operational risk management in emerging-market economies and provides a simple proposal for the construction of a database grounded on a source that has not been fully exploited yet by financial institutions\(^2\) and regulators: the insurance system. For decades, the insurance industry have gathered information about losses together with the particular circumstances those losses occurred (provided in the insurance proposal forms). Furthermore, this information is even compatible with the business lines and risk categories proposed by the BCBS. Hence, regulators have the challenge to promote the use of this information so that it becomes a net benefit for each involved party.

\(^{1}\) See FSI (2004).

\(^{2}\) Throughout the document, financial institutions in emerging markets will be understood to be financial and micro-financial institutions.
The second contribution of the paper is the construction of a complete AMA setup that hinges on multivariate analysis and includes all the Basel 2 guidelines. The method involves the calculation of the operational risk capital and its possible reduction by insurance taking\(^3\). On the other hand, an important group of international insurance companies\(^4\) have proposed a specific methodology to reduce operational risk loses due to insurance. This paper performs this methodology at face value and tests its possible implications. The paper explicitly models all the determinants of aggregate loss distributions before and after insurance, hence it allows to study the benefits of various insurance policy schemes over the final expected losses and the implied operational risk capital.

The proposed methodology represents significant advantages both for the financial and the insurance system, as well as for regulators. All of these parties, by improving their knowledge about the nature and estimation of operational risk losses under different circumstances, would be able to make better decisions.

The paper continuous as follows: Part I emphasizes the contributions of insurance in operational risk management, Part II provides analytical aspects of capital calculation under operational risk events and evaluates insurance taking. Last, the paper provides final conclusions.

\(^3\) The configuration of a methodology that consistently uses the above information within the Basel 2 principles is still in progress within the current literature.

\(^4\) See Insurance companies (2001)
Part I

Operational risk management, especially in emerging-market economies, should recognize the following contributions from the insurance industry: The role of loss financing, the promotion of best risk management practices, the potential to serve as an operational risk data source and its role as a supplier of analytical methods to estimate operational risk capital.

1. Loss financing through insurance - insurance as a capital alternative

Capital reserves are a valid option to face unexpected losses coming form operational risk events, however they might result expensive in terms of the implied opportunity costs. Operational risks, unlike credit or market risks, are not taken willingly by financial institutions because facing more operational risks do not mean obtaining higher returns. It is only desirable to hold the necessary capital to face expected losses. Then, it is sensible, to allocate a certain low amount of capital (the insurance premium) and transfer the risks of unexpected operational losses to a specialized sector (the insurance industry). Therefore, the financial system can be supported by the capital of the insurance system.

The use of insurance is of primary importance in the case of operational risks within the financial sector. This is because the majority of loss events financial institutions face are independent from each other among financial institutions, especially losses due to personnel, processes and technology. These are particular risks, namely, they correspond to idiosyncratic features of financial institutions, and hence they do not have to occur simultaneously in all the banking system. As a result, it is not expected that all the system will be affected by the realization of these types of risks in the same period of time. This implies that each bank should not be required to reserve capital for the same risks during the same period because not all of them will be affected by these risks.
Hence, the benefit to the banking system arising from the transfer of risks to insurers is clear: financial institutions face only the premium payment and allocate the remaining capital to more return-efficient uses. In conclusion, insurance creates value, and thus, it is considered as an alternative to capital.

This fact is crucially important in emerging-market economies because their financial institutions are not large enough to disregard insurance as big international financial institutions in developed economies would do. Furthermore, local insurance companies in emerging markets transfer risks to high-rated international reinsurance companies\(^5\) enhancing their financial capacity.

2. Insurance as a catalyst of good risk management practices

Insurance benefits appear not only in terms of being an alternative to capital. One of the most important functions is that insurance works, in practice, as a catalyst of good practices in operational risk management and internal control. This fact has been overlooked by many regulators and even by risk managers within financial institutions.

Effective insurance coverage not only requires the timely payment of premiums but also the commitment to comply with specific requirements set by (re)insurers. Among them, insured financial institutions must fulfill international standards of security measures and must follow specific procedures as well as sound internal auditing principles. In many cases (re)insurers require specialized audits to review some operational practices and to provide recommendations leading to promote gradual convergence to international standards. So, they indirectly promote good risk management and internal control practices within insured financial institutions\(^6\). This process and the relationship between the involved parties are close and concrete, in effect; the insurers and reinsurers backing them become a type of shadow supervisory authorities.

\(^5\) This is a standard condition set by regulators in these countries.

\(^6\) So, insurance not only contributes to the reduction of severity loss levels as is broadly recognized, but also have effects upon the loss probability because the introduction of best-practice risk management reduces the frequency of loss events.
Insurers require insureds to follow the recommendations and to take actions corresponding to observations raised by international auditing companies. The auditing serves two purposes, the verification of the information given in the insurance proposal forms and the determination of premiums based both on the specific information contained in those proposal forms and the follow-up of the recommendations.

Following the idea of Leddy (2003), the analysis made by insurers about the risk exposure of a bank to calculate the premium is equivalent to the analysis performed by a bank risk manager to calculate the economic capital of a bank and analogous to the analysis made by regulators to determine the corresponding regulatory capital.

3. Insurance as a source of data for operational risk

Some authors have proposed models for the estimation of operational risk capital, its analysis and management. Nevertheless the main weakness of these methodologies is the unconditional fashion in which operational-risk-loss data are gathered at several financial institutions. This is because the use of operational risk losses for the estimation of capital charge has ignored the conditions surrounding the occurrence of those losses due to the scarcity of loss data and the lack of data reflecting the conditions in which those losses realized. In some circumstances, the conditional factors are incorporated only at a later stage of scenario analysis, not during the capital estimation. It is important to recall that BCBS guidelines state the need of data describing the conditioning factors surrounding the loss events in the operational risk capital charge estimation.

Crucially, information about losses and their conditioning factors have been collected for decades by the (re)insurance international industry by means of insurance proposal forms. These forms contain nearly complete risk profiles of insured parties that allow estimating the premium of specific insurance contracts. Furthermore, proposal forms bear the same standards at the

\[\text{Control levels, applied technologies, complexity of operations, business lines among other conditions.}\]
\[\text{The qualitative (controls, tools, technology, practices, among others) and quantitative (level of assets, income, number of transactions, mean value of transaction, securities in transition, losses of the past three to five years, among others) conditions of the financial institutions at the time the insurance policy is taken.}\]
international\textsuperscript{9} level because insurers and reinsurers share risks requiring similar information, therefore standardized practices are almost the general norm. Some of the standard insurance proposal forms existing in the world are those of Lloyds of London. These proposal forms have been adopted by many insurance companies all over the world.

Therefore, insurance proposal forms are a data source of crucial importance for regulators, especially in emerging markets where the existing data sources such as that of the ORX are not suited for financial institutions in emerging-market economies but only for their large financial institution members. This demands regulators (particularly those with a supervisory scope in both banking and insurance sectors) to have a leading role. After all, it is not enough to unconditionally take the historical data and forecast potential financial losses and the required capital for a specific bank. It is also necessary to know the sources of those potential losses and the particular conditions those losses occurred.

Given that the implementation of Basel 2 and the use of AMA to loss modeling are planned for the coming years even in emerging market economies, their regulators should promote the use of available information potentially provided by the international insurance industry through an international data center or a similar arrangement. By doing this, they can generate relevant indicators for each risk, so that they can be included in the estimation of operational risk capital. And, it would provide the right incentives to financial institutions towards improving operational risk management practices (measured by the generated indicators) and to benefit from the implied reduction of capital charge.

4. Insurance as a provider of methods for operational risk capital estimation

Measurement of operational risk requires to know the stylized facts of the data describing operational losses. Data gathered in a systematic way and covering a reasonable period of time is scant. Nevertheless, from the few data sources currently available Embrechts et.al (2004) extracts some key features:

\textsuperscript{9} Usually reinsurers have demanded a minimum standardized information to the insurance and reinsurance brokers through insurance proposal forms (which are part of the insurance policy).
Once in a while, operational risk losses can reach extreme levels.

The frequency of operational loss events is not regular over time.

There is a seeming source of selection bias in data collected through time because financial institutions have been clustering more operational risk events in recent times as internal controls have been put gradually in practice.

Hence, the construction of statistical models to measure operational risk, have to – at least – capture those stylized facts. The aim of these models is to have reasonable quantitative approximations to the operational loss data and to use these approximations to gather more insight about the risky events under study.

Operational risk losses have features that are similar to loss events in the insurance industry. This is the reason why the existent operational risk models have much to inherit from actuarial sciences. Before introducing these models, it is useful to bring in formally the object under study in this paper: Operational risk losses.

$$S_t = \sum_{i=1}^{R} \sum_{k=1}^{n_{t,i}} z_{k}^{i} = \sum_{i=1}^{R} S_{t,i}, \quad t = 1, \ldots, T$$

Here, $S_t$ is the aggregated loss in each $t$ years in the sample. Total losses are determined by the sum of the loss events occurred in each of the $R$ categories of operational risks. The losses due to each of the risk categories $i$ in turn is the sum of all the $n_{t,i}$ observed losses in year $t$. Hence, the loss $k$ in year $t$, for operational risk of category $i$ is denoted by $z_{k}^{i}$.

With equation (1) at hand, AMA can be used to determine probability distributions for total losses or by risk categories. Once the distributions are estimated, various risk metrics (and therefore capital requirements) can be calculated easily. In general there are three AMA approaches that can be of use within this paper. The Loss Distribution Approach (LDA), Extreme Value Theory (EVT) and Ruin process models. The following subsections briefly explain these approaches and extract their benefits in view of the main purpose of this paper.
4.1 The Loss Distribution Approach (LDA)

The tasks here is to estimate parametric distribution functions for the frequency of events \( (n_{i,j}) \) and for the severity of each event \( (z_{i,j}) \) in separate ways. Afterwards, convolution operations\(^{10}\) are done to derive the total loss distribution as defined in equation (1). This allows to calculate usual risk metrics involved in the calculation of operational risk capital.

The estimation of these parametric distributions can be performed by maximum likelihood methods. Additionally, some of the usual distributions can be modeled in a multivariate fashion and hence provide a way to introduce conditioning factors for frequency and severity models\(^{11}\).

4.2 Extreme Value Theory (EVT)

In contrast to LDA models that use all available data, EVT models only consider extreme-valued data. It is sensible to think that extreme losses are more relevant to determine exposure to operational risk. Cruz (2002) points out that the key question is how much capital a bank needs to be protected of an eventual operational catastrophe. This means that for risk management purposes, the behavior in the tail of the loss distribution is of paramount interest. This is done by EVT\(^{12}\).

To apply EVT, first some metric has to be established to determine what an "extreme" value is. Once this value is known, the Peak over Threshold (POT) method is used to fit the EVT model for the selected data. The drawback of EVT models is that there are no references to estimations considering multivariate factors. This turns out to be crucial for the modeling choice in the paper because a multivariate context is preferred in order to be as close as possible to BCBS guidelines. The statistical theory of EVT has not gone too far yet to be able to condition the behavior of extreme-valued losses to diverse conditioning factors.

\(^{10}\) This method is used in the actuarial mathematics for non-life insurance.

\(^{11}\) The second part of the paper shows how these multivariate modeling can be done.

\(^{12}\) This methodology is used in the insurance industry to measure catastrophic risk events.
4.3 Ruin theory

To ease the analysis, all risk categories can be aggregated, so equation (1) can be re-written as:

\[ S_t = \sum_{k=1}^{n(t)} Z_k, \quad t \in [0, t] \]  

(1)'

Where \( n(t) \) is the number of total losses occurring in period \([0,t]\) across all risk categories\(^{13}\). Ruin theory considers the following stochastic process\(^{14}\):

\[ R_t = k + \rho t - S_t, \quad t \in [0, t] \]  

(2)

This process is defined for an initial capital amount for operational risk, \( k \) and a premium \( \rho > 0 \). Just like in insurance, the idea is the suitable choice of initial capital to avoid ruin during a given period of time. Namely, the analyst wants to know if \( R_t \) can achieve a low value (typically below zero).

Hence, operational risk capital can be directly calculated if we first define a ruin probability \( (e) \) and an interval of time \([T_1, T_2]\). Given process (2), the ruin \((R_t<0)\) probability at \( e \), is:

\[ \text{Prob}\left( \inf_{T_1 \leq t \leq T_2} (k + \rho t - S_t) < 0 \right) = e \]  

(3)

Also, in order to estimate the probability in (3), it is necessary to know the probability distribution governing \( S_t \). This can be done through LDA methods. Hence, instead of two distinct approaches, LDA and Ruin theory are rather complementary to each other.

In a nutshell, the purpose of this paper is to analyze the introduction of insurance in the estimation of net operational risk capital requirements. The paper does not need to rely on ruin theory to perform the exercise because LDA is the direct and most adequate way\(^{15}\). Neither does it use EVT models because conditioning on control factors is a difficult task within EVT. So, direct LDA methods allow easy-to-implement procedures to condition on control factors. Furthermore, loss distributions (ex-ante and ex-post to the consideration of insurance) are easily comparable.

\(^{13}\) This technique is widely used by insurers to assess the outcomes of insurance policies along the whole duration of the contract.

\(^{14}\) Also known as ruin process.

\(^{15}\) The benefit of the ruin theory approach is the study of multi-period contracts or the "long-run" capital calculation. This paper only studies one-period insurance policies.
1. The methodological proposal

Assuming the availability of data about losses and their conditional factors provided by the insurance industry, this section proposes a methodology, based on multivariate analysis, to determine operational risk capital. Then it introduces insurance considerations for capital reduction.

1.1 Estimation of initial operational risk capital

The standard LDA procedure to calculate operational risk capital has three parts: frequency, severity and total loss modeling.

Frequency

Frequency is a random variable defined as the number of loss events occurring in each period due to a specific cause. The procedure outlined here hinges on modeling the probability distribution of the number of loss events \( (n) \) conditional on a set of explanatory variables \( (X) \). These explanatory data is comprised for instance by control indicators that measure the availability and the effectiveness of controls linked to risk categories and to the business lines under study during the period of analysis. The complete data is fit to plausible parametric distribution functions on a multivariate sense\(^{16}\).

Severity modeling

The severity of losses is a random variable defined as the financial loss when each specific risk event hits. Data collected from the insurance industry contain these monetary losses \( (z) \) together

\(^{16}\) The estimation can be done by Maximum Likelihood Methods.
with the conditional variables of loss severity (Y) for each loss event\textsuperscript{17} obtained from the insurance proposal forms. The LDA literature provides tools to assess monetary loss data in terms of plausible continuous probability density functions. This paper proposes to include both (z,Y) variables and then to perform conditional estimations.

**Aggregate losses**

During a period of time, total losses are given by the sum of each event loss in the period of analysis.

\[
S = \sum_{i=1}^{n} z_i
\]

Where \( n \) is the number of events by period (frequency) taking values in the set \( \{0,1,2,...\} \). The variables \( z_1, \ldots, z_n \) define the loss severity amount of each event.

The aggregated loss \( S \) combines two random variables (\( n \) and \( z_i \)) whose conditional distributions are previously estimated. Therefore the aggregation \( S \) is itself a random variable whose distribution has to be determined by convolution methods.

The frequency of loss events (\( n \)) has an estimated probability distribution denoted by \( p_n = \Pr(N = n) \) while the loss severity distribution (\( z \)) has estimated density distribution and cumulative distribution functions denoted by \( f_Z \) y \( F_Z \), respectively. According to Bee (2005) the cumulative probability distribution function of \( S \) is defined as:

\[
F(S) = \Pr(\omega \leq S)
= \sum_{n=0}^{\infty} p_n \Pr(\omega \leq S \mid N = n)
= \sum_{n=0}^{\infty} p_n F_Z^n(S)
\]

Where \( p_n = \Pr(N = n) \) and \( \Pr(\omega \leq S \mid N = n) \) is the probability to have a cumulative loss lower than \( S \) given that \( n \) events have occurred.

\textsuperscript{17} In the practical applications, it can be necessary to carry out monotonous transformations to the conditional factors; in order to be able to derivate a function of multivariate continuous distribution without changing the information.
Due to the fact that $S$ is the cumulative sum of loss severities, its distribution functions can be estimated by the convolution procedure outlined in Appendix A. Once this is done, expected losses as well as risk metrics can be extracted. In particular, operational risk capital is such that it covers the unexpected losses for the period capital reserve was made. Unexpected losses are defined as the difference between a high percentile of the loss distribution (According to Basel 2, 99.9-percentile) and the expected loss value.

It is important to highlight that the estimation of tail percentiles is always uncertain because it depends on the specific form of loss distribution and the type of data under consideration. Therefore operational capital estimation must account for this uncertainty in the form of operational capital intervals.

### 1.2 Operational risk capital under the presence of insurance

Introduction of insurance requires first the consideration of qualitative guidelines established by the BCBS, as well as the inclusion of particular insurance contract features.

Insurance contracts change the form of the original loss distribution $F(S)$ because net losses with insurance are lower than those obtained in the absence of insurance. Following Insurance Companies (2001), the standard elements of insurance can be summarized in Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{ev}$</td>
<td>Specific deductible to each loss event</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Aggregate deductible</td>
</tr>
<tr>
<td>$l_{ev}$</td>
<td>Limit for each event of loss</td>
</tr>
<tr>
<td>$L_u$</td>
<td>Aggregate limit</td>
</tr>
<tr>
<td>$CR_p \in (0, 1)$</td>
<td>Credit risk haircut of the insurer$^{18}$</td>
</tr>
</tbody>
</table>

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$^{18}$ Even though the BCSCB establishes a minimum rating of A for insurance providers, the paper considers lower-rated insurers not reaching vulnerable levels. This is because it is too stringent to require emerging-market financial institutions to operate with insurers that rank better than their own countries. For this reason, it is necessary to perform an adjustment according to the paying ability rating of the insurance company.
The interpretation of these variables is as follows:

- If a loss event of size \( z_i \) occurs, the loss payment by the insurer is: \( z_i - d_{ev} \). This payment can not be higher than the limit \( l_{ev} \).

- If there are \( n \) loss events in the insurance period, the total sum of amounts that the insurer pays must be higher than the aggregate deductible \( D_a \) but lower than the aggregate limit. \( L_a \). When this occurs the insurer pays out an amount in excess of \( D_a \). Otherwise, the loss payment is equal to the maximum coverage \( L_a \).

- This means that cumulative high losses are effectively insured while the small cumulative losses are absorbed by the insured bank.

- The insurance effect under the features of the contract outlined above is then obtained using the following formula:

\[
S_{evg} = \sum_{i=1}^{n} z_i - CR_p \ast \min \left\{ L_a, \max \left\{ \sum_{i=1}^{n} \min \left( l_{ev}, \max (z_i - d_{ev}, 0) \right) - D_a, 0 \right\} \right\}
\]

(5)

- According to these equations, cumulative losses \( (S_{evg}) \) with insurance are lower than those in the absence of insurance \( (S) \). Therefore, the loss distribution function shifts to the left as described in Figure 1. This insurance contract allows the 99-percentile to reduce more than the expected loss value and so lowers operational risk capital.

**Figure 1: Comparison of the accumulative loss distribution with and without insurance.**
1.3 Advantages, drawbacks and importance of the methodology

Unlike other models, the proposed methodology considers conditional indicators of severity and frequency of events to model operational risk capital and introduces insurance effects upon this capital. Thus it allows to assess the marginal benefits of different conditions affecting financial institutions as well as the environment surrounding the financial system. On the other hand, this paper identifies the validity of insurance contract schemes proposed by insurers to financial institutions.

However, the current applicability of the methodology has limitations. First, the proposed source of data collection only contains information about financial institutions who take insurance\(^\text{19}\). Second, the paper does not establish the variables to be considered for each risk in a standard way, this issue deserves future research attention. The paper only proposes that the data must be built from insurance industry. Last, the inclusion of insurance in the LDA method responds to a traditional insurance scheme. Alternative risk transfers schemes demand further adjustments to the methodology.

The proposed methodology is important for financial institutions, insurers and regulators alike. It allows to capture individual bank conditions for the estimation of operational risk capital and to know the relevant information useful to make investments decisions regarding the improvement of factors with higher marginal impacts on the frequency or severity of operational losses. In the case of insurers, it provides systematic information for both the estimation of premiums and the development of insurance schemes according to individual bank situations. In the case of regulators, besides demanding them the challenge to undertake the loss data collection proposed here, it offers them a reference to assess AMA estimations presented by financial institutions.

\(^\text{19}\) In some countries financial institutions are required to take certain insurance policies. In emerging-market economies, regulators can provide incentives to financial institutions (especially in micro-finance) to use insurance given the advantages they bring as a catalyst of good risk management practices. Through this, the universe of financial institutions that take insurance would be expanded.
2. **An application to a retail bank in an emerging-market economy**

The paper concentrates on a hypothetical retail bank that operates just one business line (retailer bank) and faces just one risk category (for instance fraud and internal theft). Furthermore, the insurance policy covers only that risk. These are simplifying assumption to show how the methodology works. As mentioned before, this methodology assumes the availability of operational loss data and the conditional factors behind.

The assumption is that the operational risk controls of this bank are below the industry standard and that conditions surrounding the bank are indeed risky. This financial institution has assumed the expected losses as part of its normal controls, hence the operational risk capital to be calculated is the difference between the 99.9 percentile and the expected value of total losses. All the data employed in the analysis herein is simulated.

### 2.1 Operational risk capital without insurance

Following the previous Section, the explanatory variables of fraud and internal theft consider two types of controls: Control 1 is a control quantity indicator and Control 2 is a control effectiveness indicator. The two controls have a negative relationship with the probability of occurrence of loss events. Both control indicators are measured by percentages between 0 and 100. Table 2 summarizes the data.

<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1.92</td>
<td>1.6</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control 1</td>
<td>31%</td>
<td>19%</td>
<td>2%</td>
<td>80%</td>
</tr>
<tr>
<td>Control 2</td>
<td>34%</td>
<td>19%</td>
<td>2%</td>
<td>79%</td>
</tr>
</tbody>
</table>

**Frequency modeling**

In this application, the data is fit to a conditional Poisson distribution, namely the probability of the frequency of events is driven by
$$f(n) = \frac{\exp(-\lambda)\lambda^n}{n!}$$  \hspace{1cm} (6)

Where \( n \) represents the number of loss events due to fraud or internal theft observed every year. It is important to note that the Poisson model is conditional. Namely, every \( n_i \) observed in the sample is randomly obtained from a Poisson distribution with parameter \( \lambda_i \), which in turn, is conditioned to the two level of controls \( X^{(1)} \) and \( X^{(2)} \) of this specific example. According to Greene (1993), \( \lambda_i \) can be modeled as \( \ln(\lambda_i) = \beta' X_i \) and the parameter \( \beta \) can be obtained via maximum likelihood methods. For the present exercise, the estimation is

$$E[n_i | X_i] = \lambda_i = \exp(1.74 - 1.03 X^{(1)} - 2.69 X^{(2)})$$  \hspace{1cm} (7)

The important advantage of this formulation is that it can condition the frequency distribution to various control values. In Figure 2 for example Control 1 is set to its mean value and Control 2 is allowed to vary between 0 and 1 (poor to best-practice levels). The result is a joint probability distribution of the number of events (\( n \)) and Control 2. The lower the quality of controls, the higher is the probability of occurring more than three loss events. Conversely, when controls are close to best-practice, the probabilities are highly concentrated around zero loss occurrences.

**Figure 2: Conditional Bivariate Distribution (Frequency and Control 2) conditional on Control 1**
Severity modeling

Hypothetical loss severity amounts were simulated together with three determining key factors: net incomes, the level of complexity of business line operations and level of personnel experience in the management of the activity subject to risk. Net incomes ($NETY$) are positively correlated to the severity of events. The business complexity level ($COMPLEX$) is a continuous index between 1 and 5. The higher the business complexity is, the higher the possible severities are. The experience level ($EXPR$) varies in the continuous interval 1 to 4. The higher the experience level is, the lower the severities are. Table 3 summarizes the simulated data.

| Table 3: Descriptive statistics of severity of loss and constraint variables |
|------------------------|--------|----------------|---------------|---------------|---------------|
|                        | Nomenclature | Mean          | Standard deviation | Minimum     | Maximum       |
| Amount of losses $^a/$ | z       | US$ 77        | US$ 41           | US$ 19       | US$ 355       |
| Amount of income $^a/$ | $NETY$  | US$ 1 528     | US$ 1 149        | US$ 109      | US$ 9 500     |
| Level of complexity   | $COMPLEX$ | 3.0           | 0.8              | 1.2          | 4.8           |
| Level of Experience   | $EXPR$   | 3.2           | 0.6              | 1.2          | 4.0           |

$^a$ The amounts are expressed in hundreds of thousands.

A straightforward way to model the severity loss distributions conditional on the risk indicators outlined in this example is the multivariate Log-Normal. In this distribution, the logarithm of loss amounts together with suitable transformations of key risk indicators can be model as a multivariate normal.

Given the conditional nature of this procedure, effects like those presented in Figure 3 can be shown. There, the distribution on the left depicts a low-risk situation (benchmark) with $NETY = 1\, 200$ US$^2$ thousands, $COMPLEX = 2.5$ and $EXPR = 3.5$. The one on the right corresponds to a relatively high-risk situation of the hypothetical bank under study here ($NETY = 2\, 000$ US$^2$ thousands). Of course more interesting multivariate distributions can be used for severity modeling. For the purposes of this paper multivariate normal are enough. To model all the variables as normals, the original data is monotonically transformed (Appendix B contains the details).
thousand, \( \text{COMPLEX} = 4, \ \text{EXPR} = 2 \). In the low-risk situation, if there is an adverse event, the mean amount of loss is US$ 58 thousand; whereas in the high-risk case the mean increases to US$ 148 thousand.

Figure 3: Conditional loss severity distributions

Aggregate Loss

At this stage it is important to recall the two components of the aggregate loss distribution modeling:

- The frequency of loss events is modeled via a conditional Poisson distribution

\[
f(n; \lambda) = \frac{\exp(-\lambda)\lambda^n}{n!}
\]

With \( \lambda \) conditional to two control variables: Quantity and quality of controls.

\[
\lambda = \exp(2.26 - 1.78X^{(1)} - 3.57X^{(2)})
\]

- The severity of loss events is modeled via a Log-Normal multivariate model. Namely, the logarithm of loss amount \( x^{(0)} = \ln(z) \) is distributed according to a normal distribution conditional on risk indicators such us net income, business complexity levels and personnel experience\(^{21}\).

\[
f(x^{(0)} \mid x^{TP}) = N\left(u_{0|p}, \sigma_{0|p}^2\right)
\]

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\(^{21}\) See Appendix B for details of the log-multivariate modeling.
The convolution methodology of the conditional Poisson model for frequency and conditional log-normal for severity is based on the steps:

1. Establish the numerical value \((X^{(1)}, X^{(2)})\) to determine the value of the parameter \(\lambda\) to be used in \(f(n; \lambda)\)

2. Establish the numerical value of severity factors: net income \((NETY)\), complexity \((COMPLEX)\) and experience \((EXPR)\).

3. Generate a random number \(n\) from the frequency distribution \(f(n; \lambda)\)

4. Given such random number, obtain \(n\) values of \(x_1^{(0)}, \ldots, x_n^{(0)}\) from \(N(u_{i|p}, \sigma_{i|p}^2)\), then apply antilogarithms to obtain the losses and the sum \(S = \sum_{i=1}^{n} z_i\)

5. Repeat steps 3 and 4 a high enough number of times (for instance \(N_{sim} = 100\,000\)).

6. Depict the resulting empirical histogram for \(S\).

Again, conditional exercises can be performed. For example, in Figure 4 a low-risk and good-control situation is depicted. The total loss distribution is bi-modal, with modes at zero and at a value near US$ 58 thousands. The higher probability mass occurs at zero loss. Namely, there is high probability of losses not occurring at all because the prevailing controls are good, and when such events occur the severity of those losses are relatively low because the risk factors are relatively favorable.

Figure 4: Total loss distribution with good controls and low-risk factors
Figure 5 displays a converse situation. Namely, the case is characterized by the presence of bad controls and high-risk severity factors. In this case, up to 5 loss events occur with probability higher than zero. When these events occur, the probability to have strong losses is high. The resulting total loss distribution is asymmetric with the right tail being relatively wide.

The resulting operational risk capital

Given the density and cumulative distribution functions of aggregate losses $f(S)$ and $F(S)$ respectively and given the pre-specified extreme percentile $\alpha$ of these distributions (for example $\alpha=0.999$) the operational risk capital is the difference between the percentile $\alpha$ of the losses distribution and the expected value of losses ($ORC_{i,\alpha} = F^{-1}(\alpha) - E(S)$). For the two situations depicted in the previous section we have

<table>
<thead>
<tr>
<th></th>
<th>$ORC_{i,0.999}$</th>
<th>$ORC_{i,0.99}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situation 1 (good controls, low-risk indicators)</td>
<td>186</td>
<td>120</td>
</tr>
<tr>
<td>Situation 2 (bad control, high-risk indicators)</td>
<td>1147</td>
<td>795</td>
</tr>
</tbody>
</table>

As expected, the second situation requires a higher capital requirement due to the specific risky situation surrounding the case.
2.2 The Introduction of insurance

In order to study the effect of different insurance agreements, two cases are put forward. Scheme “A” has a structure that results useful to the bank: “the bank, given its specific features, can obtain insurance payments”. Conversely, scheme “B” contains a structure of little use to the bank: “the bank never obtains insurance payments”. This situation might describe a bank that bought insufficient insurance cover for its particular conditions.

Table 5: Parameters of possible configurations insurance schemes

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Scheme A</th>
<th>Scheme B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{we} )</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>( D_a )</td>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>( L_a )</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>( L_u )</td>
<td>1 127</td>
<td>1 127</td>
</tr>
<tr>
<td>( CR_{\alpha} \in (0,1) )</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The convolution procedure to obtain aggregate losses now takes into account the formula (5) introduced in the previous section. The procedure follows these steps:

1. Define the values for the parameters of the insurance agreement (either A or B)
2. Generate \( n \) from the Poisson function (bad control situation)
3. Generate \( n \) losses \( z \) of the Log-Normal distribution (low-risk situation)
4. Estimate \( S_{seg} \) according to equation (5)
5. Return to step 2 and repeat NSim (100 000) times
6. Obtain the expected values and \( \alpha \)-percentiles of generated distribution \( F_{seg(S)} \).
7. Estimate the implied operational risk capital with \( ORC_{f,\alpha} = F_{seg}^{-1}(\alpha) - E(S_{seg}) \).

Table 6 summarizes the results. In the “favorable” insurance scheme (A), the 99,9 percentile is reduced by US$ 212 thousand whereas the value of the expected mean loss falls by US$ 52 thousands. This means that the operational risk capital reduction is about US$ 160 thousands. On the other hand, under insurance B, the differences between the ex-ante and ex-post
insurance cases are not very distinct. Under this scheme the operational risk capital falls only by US$ 16 thousands. In practice, these small effects are not significant enough\(^2\).

### Table 6: Operational risk capital before and after the insurance

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Before the insurance</th>
<th>Insurance Scheme A</th>
<th>Insurance Scheme B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentile 99,9 %</td>
<td>1 643</td>
<td>1 431</td>
<td>1 627</td>
</tr>
<tr>
<td>Percentile 99 %</td>
<td>1 290</td>
<td>1 135</td>
<td>1 286</td>
</tr>
<tr>
<td>Expected value</td>
<td>495</td>
<td>443</td>
<td>495</td>
</tr>
<tr>
<td>(\text{ORC}_{0.999})</td>
<td>1 148</td>
<td>988</td>
<td>1 132</td>
</tr>
<tr>
<td>(\text{ORC}_{0.99})</td>
<td>795</td>
<td>692</td>
<td>790</td>
</tr>
</tbody>
</table>

Note: the values are estimated with a Monte Carlo simulation of 100 thousand repetitions.

As a conclusion, the introduction of insurance in operational risk capital estimation may have different results depending on i) the conditioning factors and environment surrounding financial institutions and ii) the particular insurance schemes taken by these institutions. The proposed methodology has precisely provided a useful tool to assess the relative benefits of the aforementioned factors.

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\(^2\) In some situations this may imply that the effect of insurance might be lower than its costs.
Conclusions

This paper emphasizes the importance of insurance in the estimation of operational risk capital and the management of operational risk. Four features of insurance have been highlighted as key to operational risk: its loss-financing role, its promotion of best risk management practices, its potential to supply a rich operational risk data source and its role as a provider of analytical methods to estimate operational risk capital.

Operational risk capital charges might turn out expensive because operational risks have the nature of being pure risks which give no return to its bearer. So, operational risk insurance covering unexpected losses become a good alternative to operational risk capital.

Insurance on the other hand, also functions as a catalyst of good practices in operational risk management and internal control. The insurance and reinsurance businesses are guided by standardized principles that require - financial institutions that take insurance - not only the timely premium payments; but also, the fulfillment of specific commitments regarding sound risk management practices.

This paper proposes the use of information that the international insurance industry have collected over decades. The advantages of these data source are considered to be beneficial for financial institutions, insurers and regulators. This is essential in emerging markets because many of their financial regulators will freely adopt Basel 2 guidelines in the future and will accept the use of AMA; and unlike developed economies, these countries do not have access to comprehensive and systematic external data sources. So regulators in these countries might find beneficial to promote the existence of an international data center based on the yet unexploited insurance system data.

In the second part of the paper, a comprehensive LDA estimation is carried out in order to compare the effects of insurance under different schemes. A key feature of the approach is the multivariate fashion in which both, the frequency of losses and the severity of each loss, are modeled together with the conditioning factors. This is the type of data that might be available with the use of the data set proposed above. The implication of the multivariate modeling
introduced in the paper overcomes the usual limitations of standard LDA approaches regarding the use of conditioning factors in the determination of operational risk capital as BCBS guidelines suggest.

Marginal changes in controls and business conditions do in practice have effects on the frequency of losses and their severity. The multivariate approach taken here allows to assess those effects. Furthermore, whether a particular insurance scheme, given specific controls and business conditions, is suitable or not, is also a question that is naturally answered in the proposed methodology. Due to all these reasons, the paper provides a useful decision making tool for financial institutions.

In particular, regulators can use the methodology as a reference for the assessment and approval of the implementation of AMA proposals. This is crucial especially when financial institutions holding insurance seek the approval of capital reduction. As observed in Part II, insurance reduces expected and non expected losses only insofar as appropriate insurance policy schemes are taken (level of deductibles, limits and son on).

Therefore, from all the issues raised here, it is possible to conclude that the insurance industry has a great potential to contribute to the management of operational risk in emerging-market financial institutions. The realization of this potential not only depends on those financial institutions but also on their regulators that may speed up the utilization of insurance services in operational risk management.
References

5. Greene, William; *Econometric Analysis* (Second Edition); Prentice Hal; EEUU; 1993.
Appendix A: A brief introduction to convolution of functions

Given two random variables \( z_1 \) and \( z_2 \) with cumulative distribution functions \( F_1 \) and \( F_2 \) respectively and a third variable defined as the sum of the previous two random variables \( y = z_1 + z_2 \), then the distribution function of the sum \( y \) is defined by

\[
F_y(y) = \int_{-\infty}^{\infty} F_2(y - z_1) F_1(z_1) \, dz_1.
\]

For instance, in the case of a summation of two random variables such as this case \((n = 2)\), the sum is \( S = z_1 + z_2 \) with distribution

\[
F_S^2(S) = \int_{0}^{\infty} F_Z(S - z) F_Z(z) \, dz.
\]

Then for the case with \( n = 3 \) the sum is \( S = z_1 + z_2 + z_3 \) and the previous result can be used to obtain:

\[
F_S^3(S) = \int_{0}^{\infty} F_Z^2(S - z) F_Z(z) \, dz.
\]

So, given an arbitrary \( n \), the distribution function of the sum can be recursively obtained. Therefore, equation (4) in the main text appears as a convolution of order \( n : F_S^n(S) \)

Appendix B: Severity modeling

B1 Transformation of variables in the severity model

The amount of losses \( z \) are transformed according to \( x^{(0)} = \ln(z) \). Net income \((NETY)\) is transformed through similar expression \( x^{(1)} = \ln(NETY) \). The business complexity variable \((COMPLEX)\) takes continuous values on the interval \([1,5]\), in order to model complexity as a normal distribution the following change is proposed \( x^{(2)} = \Phi^{-1}((COMPLEX - 1) / 4) \), namely the range is shifted to the interval \([0,1]\) and then inverse cumulative standard normal is applied to obtain values \( x^{(2)} \in (-\infty, \infty) \). The experience variable \((EXPR)\) has a similar treatment: \( x^{(3)} = \Phi^{-1}((EXPR - 1) / 3) \).
B2 Multivariate severity modeling

In the estimation of the severity distribution, the normal distribution function contains the set of previously transformed severity and explanatory factors. \( x = \begin{bmatrix} x^{(0)} & x^{(1)} & \ldots & x^{(p)} \end{bmatrix} \in \mathbb{R}^{p+1} \)

So, the normal distribution can be expressed as:

\[
f(x) = (2\pi)^{\frac{1}{2}(p+1)} |\Sigma| \exp \left( -\frac{1}{2} (x - \mu)^\top \Sigma^{-1} (x - \mu) \right)
\]

From this joint distribution, the density function for the log of severity amounts \( x^{(0)} \) conditional on the rest of the variables \( X_{pp} = \begin{bmatrix} x^{(1)} & x^{(2)} & \ldots & x^{(p)} \end{bmatrix} \) can be easily obtained. To do this, it is necessary to partition the vectors of means the variance-covariance matrix in a conformable fashion:

\[
\begin{bmatrix} \mu^{(0)} \\ \mu^{(1)} \\ \vdots \\ \mu^{(p)} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \Sigma_{00} & \Sigma_{01} & \ldots & \Sigma_{0p} \\ \Sigma_{10} & \Sigma_{11} & \ldots & \Sigma_{1p} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{p0} & \Sigma_{p1} & \ldots & \Sigma_{pp} \end{bmatrix}
\]

The conditional distribution of \( x^{(0)} \) given determining factors is:

\[
f(x^{(0)} | x_{pp}^\top) = N \left( u_{0|p}, \sigma_{0|p}^2 \right)
\]

Where:

\[
u_{0|p} = \left( \mu^{(0)} - \Sigma_{0p}^\top \Sigma_{pp}^{-1} u_{pp} \right) + \Sigma_{0p}^\top \Sigma_{pp}^{-1} X_{pp}
\]

\[
\sigma_{0|p}^2 = \sigma_0^2 - \Sigma_{0p}^\top \Sigma_{pp}^{-1} \Sigma_{pp}
\]

The unconditional mean of the risk factors is denoted by \( u_{pp} \), their covariance matrix is given by \( \Sigma_{pp} \) and the covariance between log-severity and each of the risk factors is denoted by \( \Sigma_{0pp} \).

\[
u_{pp} = \begin{bmatrix} \mu^{(1)} \\ \vdots \\ \mu^{(p)} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \Sigma_{11} & \ldots & \Sigma_{1p} \\ \vdots & \ddots & \vdots \\ \Sigma_{p1} & \ldots & \Sigma_{pp} \end{bmatrix} \quad \Sigma_{0pp} = \begin{bmatrix} \Sigma_{01} & \ldots & \Sigma_{0p} \end{bmatrix} = \begin{bmatrix} \Sigma_{10} \\ \vdots \\ \Sigma_{p0} \end{bmatrix}
\]
The parameter that defines the conditional mean \( u_{0|P} \) depends on the value of determining factors summarized in the vector \( X \):

\[
u_{0|P} = \left( \mu^{(0)} - \Sigma_{\Upsilon \Upsilon}^{-1} \Sigma_{\Upsilon \Upsilon P} u_{\Upsilon P} \right) + \Sigma_{\Upsilon \Upsilon P}^{-1} \Sigma_{\Upsilon P} X_{\Upsilon P}
\]

The parameter that defines the conditional variance does not depend on controls.

\[
\sigma_{0|P}^2 = \sigma_0^2 - \Sigma_{\Upsilon \Upsilon P}^{-1} \Sigma_{\Upsilon P} \Sigma_{\Upsilon \Upsilon P}
\]

The parameter values are:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu^{(0)} )</td>
<td>Unconditional mean of log losses</td>
</tr>
</tbody>
</table>
| \( \Sigma_{\Upsilon \Upsilon} \) | Covariance of log losses and transformed conditioning variables | \[
\begin{bmatrix}
0.08 \\
0.10 \\
-0.22
\end{bmatrix}
\]
| \( \Sigma_{\Upsilon \Upsilon P} \) | Covariance matrix of transformed conditioning variables | \[
\begin{bmatrix}
0.40 & 0.02 & 0.00 \\
0.02 & 0.32 & 0.05 \\
0.00 & 0.05 & 0.69
\end{bmatrix}
\]
| \( u_{\Upsilon P} \) | Unconditional mean of transformed conditioning variables | \[
\begin{bmatrix}
7.12 \\
0.00 \\
0.83
\end{bmatrix}
\]
| \( \sigma_{0|P}^2 \) | Conditional variance of log losses | 0.09 |