Liquidity risk and financial instability: an alternative approach to calculate the probability of bank exit

Paul Collazos*

Abstract

In contrast to usual early warning models, which require considerable evidence on bank exits, this essay shows an alternative way to calculate the probability of a bank exit if such data is not available. This essay uses a simple “perpetual selection” story as it is defined by Fudenberg and Tirole in order to offer a theoretic analysis of how liquidity shocks á la Diamond-Dybvig would affect the strategies followed by bankers, in particular their probability of exit. The bottom line suggests a new channel (different to bank runs) between liquidity risk and banking sector instability.

Resumen

En contraste con los modelos de alerta temprana tradicionales que requieren una abundante evidencia sobre quiebras bancarias, este trabajo muestra una forma alternativa de calcular la probabilidad de que un banco salga del mercado si esa información no está disponible. Este artículo utiliza un proceso de “selección perpetua”, tal como lo definen Fudenberg y Tirole (1986), para ofrecer una análisis teórico de cómo shocks de liquidez á la Diamond-Dybvig podrían afectar las estrategias seguidas por banqueros, en particular su probabilidad de salida del mercado. Finalmente se sugiere un nuevo canal (diferente a las corridas bancarias) entre el riesgo de liquidez y la inestabilidad del sector bancario.

Keywords: Bank exit, War of attrition, Liquidity risk
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* The author is a Senior Economist in the Research Department at the Superintendency of Banks, Insurance Companies and Private Pension Funds of Peru and a Lecturer of the Department of Economics at the Pontificia Universidad Católica del Perú and the Universidad del Pacífico.

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The estimation of early warning indicators for financial crises has generated an abundant literature that seeks to attain a measure of the probability of a bank exit. The first generation of these studies consisted of actuarial models adapted to calculate the probability of “death” of surviving banks. Using multivariate analysis (Sinkey, 1975), discrete choice models (Martin, 1977) or hazard models (Lane, Looney and Wansley, 1986), they sought to extract a probability of bank exits that is positively correlated with the observed frequency of bank deaths. In particular, if a country did not experience bank exits during a considerable long period, then -according to these models- the probability of a bank exit in this country would be close to zero.

The evidence of banking crises in countries where the banking sector did not suffer bank exits during many years before the crisis emerged, lead to the second generation of early warning models developed by Gonzales-Hermosillo (1996) and Gonzales-Hermosillo, Pazarbasioglu and Billings (1996). These models started with the definition of a set of financial variables for banks (usually related with the CAMEL model), which are then evaluated and compared in the period before and after a financial crisis. Finally, the authors constructed an index of banking sector fragility based on indicators that increase the probability of a bank exit and reduce the survival time of a financial institution.

The frequency of financial crises and the large number of bank deaths in emerging economies at the end of the past decade have made it possible to test and refine these models. A limitation of this type of analysis, however, is that the observed occurrence of bank exits may not reflect the actual occurrence of failure in banks. This is due to the incidence of bank mergers to replace the costly process of bank exits as well as government efforts to bail out small banks through recapitalization, governmental insurance or the absorption by a big bank.

Another disadvantage of these models is the large number of variables needed to obtain a probability of a bank exit. They usually employ seven subsets of banking indicators: two subsets of ratios (non-performing loans to total assets and capital to total assets) and five subsets of indicators for market risk, credit risk, liquidity risk, moral hazard and contagion. The empirical results demonstrate that the ratios of non-performing loans and the indicators of liquidity risk are the best determinants of bank fragility. Moreover, in the five episodes examined in Gonzalez-Hermosillo...
(1999), only liquidity risk indicators systematically show the expected results of the model: a positive impact on the probability of a bank exit and a negative impact on the survival time. This essay, thus, focuses on the link between the liquidity risk and probability of a bank exit.

This essay proposes an alternative way to calculate the probability of a bank exit using a simple example of an economy in which liquidity shocks à la Diamond-Dybvig may be the most important determinant of the probability of banks exit. I follow Diamond and Dybvig (1983) and Jacklin and Bhattacharya (1988) in terms of viewing investors as ex-ante identical individuals who live only three periods. During the first period, their types (patient or impatient) are unknown to themselves but starting the second period each investor realizes his own type. Investor’s preferences modeled in this way are a hybrid between Diamond and Dybvig’s extreme case and the more general case exhibited by Jacklin and Bhattacharya. As a result, patient investors receive utility only from third period consumption and impatient investors receive utility from second and third period consumption. In my example, production technology is exactly the one presented by Diamond and Dybvig: if the production process is interrupted before the beginning of the third period, the investor obtains a low return. But if the production process is not interrupted, then it yields a high return, usually interpreted as a long-term return.

This example differs from the models of Jacklin and Bhattacharya, and Diamond and Dybvig, when I consider the existence of agents who live infinitely (called bankers) who have access to the same technology and whose role is to alleviate the liquidity risk of investors. This is possible because bankers are risk neutral and are able to obtain expected returns during consecutive generations of investors. In this context, the bankers would initiate a "war of attrition". The rationale is as follows: if bankers follow pure strategies, Bertrand equilibrium\(^5\) arises and banks offer the contract that maximizes investors’ welfare subject to their participation constraint. On the other hand, if bankers follow mixed strategies, then offering the same contract would generate economic losses. Nevertheless, bankers will continue offering this contract because they believe others would stop first, so each bank would eventually earn monopolistic gains. This story constitutes an example applied to banking of the symmetric and stationary mixed-strategy Nash equilibrium what Fudenberg and Tirole (1986) called “Perpetual Selection”.

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\(^5\) Adao and Temzelides (1998) present the first analysis of a Bertrand Equilibrium in a Diamond and Dybvig Economy, but their work is focused on depositor’s beliefs on an eventual bank run using a two-period setup. In contrast, my analysis focuses on banker’s beliefs on an eventual exit of the market. Moreover, the equilibrium à la Bertrand used in this essay is only an intermediate step to solve the infinite horizon problem.
This kind of equilibrium is interesting because replicates the fact that there exist a positive probability of a bank exit even we do not observe an effective exit of the market. Moreover, in this kind of Nash equilibrium, banks have a computable probability of exit. In particular, this essay finds an expression for this probability in which investors’ relative risk aversion coefficient, the probability of a liquidity shock and the cost of opportunity of the banker are found as the main argument of this function.

Following this introduction, this essay is organized as follows: Section II presents the investors and describes the autarkic and optimal solutions without bankers. Section III considers the participation of bankers following pure strategies and considers the extremes cases when there is a monopoly and an oligopoly according to Bertrand conjectures. Section IV introduces the possibility of following mixed strategies. It describes the story of a war of attrition, defines a subgame perfect equilibrium and calculates the probability of a bank exit. Finally, Section V offers some concluding remarks.

In this section I consider an economy populated by infinite and consecutive generations of investors who live only three periods.

A. Preferences

Investors are a continuum of agents with total measure one. The preferences of each investor can be represented by a particular version of the generalized utility function presented by Jacklin and Bhattacharya (1988). In T=t-1, investors are identical and do not know neither their own nor the others’ type. In T=t each investor learns his own type, but does not learn about the type of others. Either he is patient and consumes only at the end of period T=t+1 or impatient and consume something at the end of period T=t and something at the end of period T=t+1. They know that an investor is impatient with probability \( \pi \) and patient with probability \( 1-\pi \). Preferences for consumption in periods 1 and 2 are represented by:

\[
   u(c_t, c_{t+1}) = \begin{cases} 
   u(c_t) + \rho u(c_{t+1}) & \text{if individual is impatient} \\
   \rho u(c_{t+1}) & \text{if individual is patient}
   \end{cases}
\]
where \( u: \mathbb{R} \rightarrow \mathbb{R} \) is twice continuously differentiable, increasing, and strictly concave, \( u(0) = 0 \) and satisfies Inada conditions \( u'(0) = \infty \) and \( u'(\infty) = 0 \).

**B. Endowments and technologies**

At \( T=t-1 \), all investors receive endowments of a capital good which when is invested yields a return in the form of the consumption good. There are two available technologies: a short-term technology that yields one unit of consumption good for each unit of capital good, and a long-term technology which produces a return \( R \) greater than 1. Capital goods are infinitely divisible, whatever portion of the production can be invested in the short-term technology, and the remaining will be automatically invested in the long-term technology. However, when investment decisions are made in period \( T=t-1 \), no changes are feasible which means that long-term capital investments are irreversible.

**C. Autarky**

In the simplest case, in which there is no trade among investors, each investor chooses his consumption profile. The representative investor maximizes his expected utility by solving the following constrained optimization problem in period \( T=t-1 \):

\[
\begin{align*}
\text{Max} & \quad \pi u(c_t) + \rho u(c_{t+1}) \\
\text{subject to} & \quad c_{t+1} = R(1 - c_t)
\end{align*}
\]

The autarkic allocation satisfies the first order condition:

\[
\pi u'(c_t) = R\rho u'(R(1 - c_t))
\]

**D. Optimal risk sharing without bankers**

If only investors are considered, there will be a unique symmetric Pareto optimal allocation \((c_t, c_{t+1})\), obtained by solving:

\[
\begin{align*}
\text{Max} & \quad \pi u(c_{1,t}) + \pi \rho u(c_{1,t+1}) + (1 - \pi) \rho u(c_{2,t+1}) \\
\text{subject to} & \quad (1 - \pi)c_{2,t+1} + \pi c_{2,t+1} = R(1 - \pi c_{1,t})
\end{align*}
\]
Note that, in this case I only consider two consumption bundles in t, one for each type of investors (c_{1,t} represents inpatient’s consumption and c_{2,t+1} represents patient consumption). The resulting optimal allocation satisfies the first-order condition:

\[ u'(c_{1,t+1}^*) = u'(c_{2,t+1}^*) = R \cdot \rho \cdot u'(R(1-\pi c_{1,t}^*)) \]

The first equality indicates that \( c_{1,t+1}^* = c_{2,t+1}^* \). When this result is replaced on the second equality the Pareto efficient solution is obtained. I borrow the idea of Wallace (1988) about a cash machine in order to imagine the mechanism of the optimal allocation when there are no bankers.

The previous section introduced one side of this model: the investors. It considered the autarkic solution and also presented the optimal solution. The key assumption to obtain this result was the existence of a cash machine that allows investors obtain the optimal allocation. However, there is not necessity for this assumption.

This section will focus on the other side of the model: the behavior of the bankers. Two assumptions are relevant: first, the cash machine is not available. Second, the bankers only can follow pure strategies. The first assumption justifies the existence of agents called “bankers”. The second assumption transforms the infinite horizon optimization problem of the bankers in a static minimization problem, which must be considered as an intermediate step towards the case with mixed strategies.

**A. One banker**

In an attempt to model banks as profit-maximizing institutions rather than as social planners, I assume that there is a monopolist who is called banker and whose unique objective is to maximize profits. In addition, I consider that each banker lives infinite periods, so he faces a new set of investors every three periods. The banker does not receive any endowment but does have access to long-term and short-term technologies. There is infinite number of potential investors every three periods.
The banker offers a contract \( \{d_t, d_{t+1}\} \) which requires an investment of one unit of the capital good in exchange for the right to withdraw either \( d_t \) in period \( t \) or \( d_{t+1} \) in period \( t+1 \). When an investor accepts this contract, his expected utility is given by:

\[
\pi u(d_t) + \rho u(d_{t+1})
\]

On the other hand, the expected per generation (each three years) profit earned by the banker is defined here as the excess return of a unit of investment over cost of the bundle offered to the investors:

\[
R \left( 1 - d_{t+1} \right) - \pi d_t
\]

Two observations about the rationality of investors and bankers are of interest. The first one is whether or not each investor is willing to acquire the bundle offered by the banker. This incentive constraint is satisfied if investors prefer this bundle rather than the autarkic allocation. The second observation is that the banker’s opportunity cost is \( V \geq 0 \). This opportunity appears one time in three years, exactly when a new generation of investors is born. The participation constraint for this banker requires that \( R \left( 1 - d_{t+1} \right) - \pi d_t \) must be higher than \( V \). Therefore, subject to the participation constraints, the allocation that maximizes the per-generation expected profits is obtained by solving the following minimization problem subject to the incentive and participation constraints:

\[
\min_{d_t, d_{t+1}} \quad R \ d_{t+1} + \pi \ d_t
\]

s.t. \( \pi u(d_t) + \rho u(d_{t+1}) \geq \pi u(c_t^A) + \rho u(c_{t+1}^A) \)

\[
R \left( 1 - d_{t+1} \right) - \pi d_t \geq V
\]

**B. Several bankers**

In this section, I present the case in which there are \( J \) bankers in the economy. All of them have identical information about the probability of each type of investor, the same opportunity cost and everyone has access to the same technology. Each banker offers a sequence of contracts every three periods \( \{d_t, d_{t+1}\} \). On the other hand, a given member of a generation of investors looks for the bank that offers a contract that maximizes his utility. When this investor finds that bank, he puts all his capital in that bank and the rest of bankers receive no financing.
The expected profit of a banker $j$ that offers a contract $\{d_i,d_{i+1}\}$ is given by the excess return of a unit of investment over cost of the bundle offered to the investors:

$$D_j = (1 - d_{i+1}^j) R - \pi d_i^j$$

However, if a banker offers a contract as $\{d_i,d_{i+1}\}$ then his profits are the cost of opportunity $V$. Finally, if all bankers offers the same contract $\{d_i,d_{i+1}\}$, then their profits are $(D_i/J)$.

In this context, a Nash equilibrium a la Bertrand is defined as both a pair of contracts $(\{d_i,d_{i+1}\},\{d_i,d_{i+1}\})$ and a pair of investment amounts $(k_i,k_{i-1})$ such that:

a) If $\{d_i,d_{i+1}\} <\{d_i,d_{i+1}\}$, then $k_i = 0$, $k_{i-1} = 1/J - 1$

b) If $\{d_i,d_{i+1}\} >\{d_i,d_{i+1}\}$, then $k_i = 1$, $k_{i-1} = 0$.

c) If $\{d_i,d_{i+1}\} = \{d_i,d_{i+1}\}$, then $k_i = k_{i-1} = 1/J$.

d) All bankers are not willing to obtain a profit smaller than $V$ (Participation Constraint).

e) All investors are not willing to obtain a level of utility smaller than the autarkic level (Incentive Constraint).

Therefore, the unique contract which allows a Nash equilibrium in pure strategies is $\{d_i,d_{i+1}\} = \{d_i,d_{i+1}\} = \{d_i^*,d_{i+1}^*\}$ which solves the following problem:

$$\begin{align*}
\text{Max} & \quad \pi u(d_i) + \rho u(d_{i+1}) \\
\text{s.t.} & \quad d_{i+1} = R \left(1 - d_i\right) \\
& \quad \pi u(d_i) + \rho u(d_{i+1}) \geq \pi u(c_i^*) + \rho u(c_{i+1}^*) \quad (IC) \\
& \quad \frac{R \left(1 - d_{i+1}\right) - \pi d_i^j}{J} \geq V \quad (PC)
\end{align*}$$

This solution is also a Bertrand equilibrium because it is the unique bundle such that if a banker offers a better contract then he incurs in net economic losses, but if he reduces his offer then the investor abandons him to take any other offer.

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6 Remember that this is an arbitrary division. In fact, any criterion could be used.
C. Example to obtain the viability condition for banking.

In this subsection, I offer a parameterized example of the model described in Section II and III. For this example, I show that the cost of the bundle offered by banks is a function of investors’ welfare in autarky. This result is important to reveal which assumptions support the viability of banking.

The only additional assumption is that \( u(x) = \frac{1}{\alpha} x^\alpha \), where \( 1-\alpha \) is the relative risk aversion coefficient. Solving the maximization problem of the autarkic case, the following expression for investor's welfare is obtained:

\[
U_a = \left( \pi + \rho \frac{1}{\alpha} \left( \frac{R}{\pi} \right)^{\alpha} \left( c_i \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}
\]

The typical assumption in this kind of models is to consider that the coefficient of relative risk aversion is smaller than one\(^7\), then \( \alpha < 1 \). With this assumption is possible to show that the level the utility of autarky \( U_a \) is an increasing function of the probability of a liquidity shock \( \pi \), of the long term rate of return \( R \) and the discount rate \( \rho \). The autarkic utility level is also an increasing function of the coefficient of relative risk aversion. The following figure illustrates the positive relationship between \( U_a \) and \( \pi \), and the negative relationship between \( U_a \) and \( \alpha \), when \( R=1.03 \) and \( \rho=1 \) are assumed.

\( ^7 \) This assumption appears since the article by Diamond and Dybvig (1983) and also in the one by Jacklin and Bhattacharya (1988) who assume \( \alpha = \frac{1}{2} \). This assumption is made in order to assure that each type never envies the optimal allocation of the other type.
The cost of the bundle offered by the banker to the investors is obtained by solving the minimization problem of the monopolist’s minimization problem:

\[ C_m = \theta U_a^{\frac{1}{\alpha}} = \theta \left( \pi + \rho^{\frac{1}{1-\alpha}} \left( \frac{R}{\pi} \right)^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\alpha}} \left( c^i \right) \]

Where:

\[ \theta = \frac{\frac{1}{\alpha}}{\left( \pi + \rho^{\frac{1}{1-\alpha}} \left( \frac{R}{\pi} \right)^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\alpha}}} \]

Thus the minimal cost of the monopolist Cm is an increasing function of the probability of a liquidity shock \( \pi \), of the long term rate of return R and the discount rate \( \rho \). The minimal cost of the monopolist is an increasing and then decreasing function of \( \alpha \). The next figure illustrates the positive relationship between Cm and \( \pi \) when R=1.03 and \( \rho =1 \) are assumed. The figure also shows that the relationship between Cm and \( \alpha \) is non monotonic.

**Figure No 2**
Values of Cm respect to \( \pi \) and \( \alpha \), When \( p=1 \) and R=1.03
Using these definitions it is possible to determine the unique necessary and sufficient condition, in terms of the parameters of the model, for the viability of banking. In this example, banking is viable if and only if \( R - \frac{\alpha}{\sqrt{\theta}} U^+ \geq V \). The rationale is as follows: When this condition is satisfied, the typical (representative) banker obtains positive expected profits and investors lend to the banker. However, if this condition is not satisfied, then the banker who offers an attractive bundle incurs in net losses.

Particularly, if \( \alpha < 1 \), \( R > 1 \) and \( V = 0 \) then the viability for banking is guaranteed in this example.

In this section, I discuss the case of several bankers that can follow mixed strategies. Basically, I introduce the possibility that bankers could stop their activities in some period. It happens under the assumption that they follow a mixed-strategy in which a banker stop at \( t \), with a probability \( p \), if the others bankers have not stopped before them. Probability of continuation is \( 1-p \).

### A. Implications on Bertrand Equilibrium

In the case of the Bertrand equilibrium among \( J \) bankers, all of them must offer an attractive bundle to investors (i.e. a bundle with a utility higher than autarky’s welfare) but also must be a bundle that yields a profit at least equal to \( V \).

Each banker has a positive probability to become a monopolist if he remains when the rest of bankers exit the industry. Then, there is an expected increase in his gains, which allows him to make eventual economic losses. This story constitutes an example of a war of attrition among bankers in which they remain in competition, even when incurring losses, because they believe that other bankers would drop out before them, so each bank would eventually become a monopolist and obtain positive gains. Fudenberg and Tirole (1986) named this kind of competence “Perpetual Selection”.

By construction, the contract from the Bertrand equilibrium \( \{d^*_i, d^*_i\} \) does not origin losses or gains to bankers. However, the new contract \( \{d'_i, d'_i\} \) must produce the maximum eventual economic loss: \( V^8 \).

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8 This new contract coincides with the previous if only if \( V = 0 \).
Let define a period $\tau$ in which an arbitrary banker became a monopolist. According to this, the expected present value of bankers’ profit until period $\tau-3$ will be negative:

$$V(\tau - 3) = \frac{1 - \rho^t}{1 - \rho^3} V$$

However, when considering the case in which a given banker becomes a monopolist in period $\tau$, then the expected present value of what bankers equals to:

$$V(1,\tau) = \frac{1 - \rho^t}{1 - \rho^3} V + \frac{\rho^t}{1 - \rho^3} \left\{ \left[ R - \theta U_a \right] - V \right\}$$

**B. Stationary, symmetric and mixed-strategy equilibrium**

Following Fudenberg and Tirole (1986), I define an unique Nash, stationary, and symmetric equilibrium, which is given by the mixed strategy that defines an exit probability $p_t = p$, conditional to the opponents not stopping previously. This probability is such that

$$V(t - 3) = pV(1,t) + (1 - p)V(t)$$

By replacing expressions $V(1,t-3)$ and $V(t)$, one obtains a formulation for $p_t = p$ given by:

$$p = \frac{V - \rho^3V}{\left[ R - \theta U_a \right] - \rho^3V}$$

Where $p$ is a number between 0 and 1, only if banking is viable. Moreover, the probability of a bank exit is inversely related to the viability condition. It is important to review some conclusions from this result. First, the probability of liquidity shocks on investors affects positively the probability of banks exit. Other factors influencing on $p$ are: the opportunity cost that affects positively, the long-term return with a negative effect, the intertemporal discount rate that affect negatively and the investors’ coefficient of relative risk-aversion both with a negative effect on $p$.

All these parameters are frequently used by financial analysts, bank supervisors, and ranking agencies and many of them can be calculated using the data that is included in bank reports. For this reason, that finding is an useful tool to approximate the probability of a bank exit when there is no enough evidence to apply the actuarial techniques used by the models of early warning.
This essay uses a simple “perpetual selection” story as it is defined by Fudenberg and Tirole (1986) in order to offer a theoretic analysis of how liquidity shocks à la Diamond-Dybvig would affect the strategies followed by bankers, in particular their probability of exit. This variable derives from a unique Nash, mixed, symmetric and stationary equilibrium in which all bankers continue competition with an implicit probability of exit, so each bank would eventually earn monopolistic profits.

In the following brief commentary, I provide two concluding remarks from this exercise. First, in contrast to usual early warning models, which require considerable evidence on bank exits, this paper shows an alternative way to calculate the probability of a bank exit if such data is not available. In particular, the example presented in this essay only requires three variables (a liquidity risk indicator⁹, a proxy of the opportunity cost of the banker and a discount rate) in order to calculate the probability of a bank exit.

Second, there is a positive relationship between the probability of a bank exit and the probability of a liquidity shock. This result is interesting because it shows a new causal channel from liquidity shocks towards banking system instability. The usual cause (bank runs) and the optimal design of its remedy (deposit insurance) have been the main concerns of most of the works since the seminal work by Diamond and Dybvig. However, this essay suggests that an increase in liquidity risk through the expansion of the demand for liquidity insurance affects positively the probability of a bank exit. This is because, some bankers remain in competition for deposits, even with economic losses, because they expect that other bankers would drop out before them, so each bank would eventually become a monopolist and obtain positive gains. In other words, bank competition appears as an alternative channel between liquidity risk and banking sector instability.

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⁹ A further analysis of the performance of different liquidity risk indicators (during several episodes of banking crises) constitutes the next step in this attempt to present an alternative approach to calculate the probability of a bank exit.


Hausmann, Ricardo and Liliana Rojas-Suarez (1996), Banking Crises In Latin America, Inter-American Development Bank.


